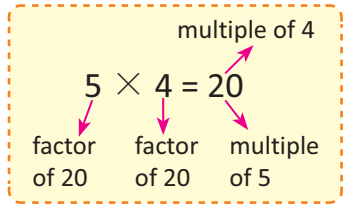
2

Factors and Multiples

Check What I Know

4 × 5 = 20 → Here 4 and 5 are _____ of 20 and 20 is a _____ of 4 and 5.



A **factor** of a number divides the number without leaving a remainder.

4 and 5 are factors of 20 because 20 ÷ **5** = 4 and 20 ÷ 4 = 5

A **multiple** of a number is exactly divisible by the number. It is obtained by multiplying the number by 1, 2, 3, 4, …, and so on.

The multiples of 5 are: 1 × 5 = **5**; 2 × 5 = **10**; 3 × 5 = **15**; 4 × 5 = **20** … and so on.

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Properties of factors

1. 1 is a factor of every number.
2. Every number is a factor of itself.
3. A factor of a number is smaller than or equal to the number.
4. The smallest factor of a number is 1.
5. The greatest factor of a number is the number itself.
6. A number has a limited number of factors. Every number (other than 1) has at least two factors—1 and the number itself.

Properties of multiples

1. Every number is a multiple of 1.
2. Every number is a multiple of itself.
3. A multiple of a number is greater than or equal to the number.
4. A number has an uncountable number of multiples. There is no largest multiple of a number.

The factors of a number can be found by multiplication or division.

Factors of 12 by multiplication

$1 \times 12 = 12$: ___ and ___ are factors

$2 \times 6 = 12$: ____ and ____ are factors

$3 \times 4 = 12$: ____ and ____ are factors

Factors of 12 by division

$12 \div 1 = 12$: ___ and___ are factors

$12 \div 2 = 6$: ___ and ___ are factors

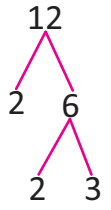
$12 \div 3 = 4$: ___ and ___ are factors

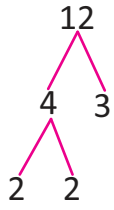
The factors of 12 are: ___, ___, ___, ___, ___, ___ *(Refer Maths Lab Activity on page 112.)*

Similarly you can find that the factors of 16 are 1, 2, 4, 8 and 16

The common factors of 12 and 16: 1, ___, ___

A factor tree shows the different ways in which a number can be split into its factors.





The first 10 multiples of 2: 2, 4, 6, ___, ___, ___, ___, ___, ___, 20

The first 10 multiples of 3: 3, 6, 9, ___, ___, ___, ___, ___, ___, 30

The common multiples of 2 and 3: 6, ___, ___, ...

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EXERCISE 1

1. **Find the first 10 multiples of each of the following pairs of numbers. Then list the common multiples.**

   a) 4, 5 b) 2, 6 c) 3, 5

2. **Find the factors of each of the following pairs of numbers. Then list the common factors.**

   a) 18, 6 b) 14, 21 c) 4, 5

Rules of divisibility

You have read about the rules of divisibility by 2, 3, 5, 9 and 10 in Class 4. The rules of divisibility help you to quickly find the factors of a number.

Rules related to units digit

A number is:

Divisible by 2 if it ends in 0, 2, 4, 6 or 8.

262, 354, 2790, 1286 and 76,858 are divisible by 2.

***Divisible by 5** if it ends in 0 or 5.*

95, 4450, 7565, 34,850 are divisible by 5.

***Divisible by 10** if it ends in 0.*

50, 430, 5670, 45,780 are divisible by 10.

Rules related to sum of digits

A number is:

***Divisible by 3** if the sum of the digits of the number is divisible by 3.*

198: 1 + 9 + 8 = 18 is divisible by 3, therefore 198 is divisible by 3.

688: 6 + 8 + 8 = 22 is not divisible by 3, therefore 688 is not divisible by 3.

***Divisible by 9** if the sum of the digits of the number is divisible by 9.*

693: 6 + 9 + 3 = 18 is divisible by 9, therefore 693 is divisible by 9.

984: 9 + 8 + 4 = 21 is not divisible by 9, therefore 984 is not divisible by 9.

Other rules

A number is:

***Divisible by 4** if the number formed by the last 2 digits is divisible by 4.*

3856 is divisible by 4 since 56 is divisible by 4.

2850 is not divisible by 4 since 50 is not divisible by 4.

***Divisible by 6** if it is divisible by both 2 and 3.*

72, 198, 4854 are divisible by 6, since they are divisible by both 2 and 3.

EXERCISE 2

1. **Put a ✓ on the numbers that are divisible by:**
   a) **2:** 21, 76, 123, 224, 5696
   b) **5:** 56, 65, 160, 745, 5551
   c) **10:** 85, 108, 940, 605, 3580

2. **Put a ✓ on the numbers that are divisible by:**
   a) **3:** 51, 73, 127, 233, 6598
   b) **9:** 98, 123, 162, 540, 9981

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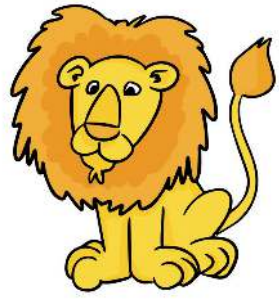
3. **Put a ✓ on the numbers that are divisible by:**

a) **4:** 146, 274, 728, 904, 6500

b) **6:** 99, 124, 426, 540, 9982

4. **Put ✓ if divisible and ✗ if not divisible.**

	Divisible by						
Number	2	3	4	5	6	9	10
a) 99							
b) 100							
c) 96							
d) 450							
e) 702							
f) 330							
g) 285							
h) 1200							



5. **Find all the factors of numbers 1–15. Use your knowledge of divisibility rules wherever possible. Remember to include 1 and the number itself. Fill in the table.**

Number	Factors	Number	Factors	Number	Factors
1		6		11	
2		7		12	
3		8		13	
4		9		14	
5		10		15	

Prime and composite numbers

Look at the table of Exercise 2 Question 5. List the numbers greater than 1 that have:

- Only 2 factors (1 and the number itself): _____

  Numbers that have only two factors are called **prime numbers.**

- More than 2 factors: _____

Numbers that have more than two factors are called **composite numbers.**

1 is neither a prime nor a composite number. It is a unique number.

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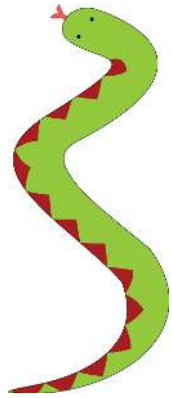
The Sieve of Eratosthenes

Eratosthenes was a Greek mathematician who lived in the third century A.D. He found a simple method of finding prime numbers. His method is known as the **Sieve of Eratosthenes**.

On the number grid, proceed as follows. Some are done for you.



1. Cross out 1.
2. Circle the number 2. Cross out every second number from 2— that is cross out all multiples of 2.
3. Circle the number 3. Cross out every third number from 3— that is cross out all multiples of 3.
4. Circle the number 5. Cross out every fifth number from 5— that is cross out all multiples of 5.
5. Circle the number 7. Cross out every seventh number from 7— that is cross out all multiples of 7.
6. Continue to circle and cross out in this manner.

   The circled numbers are the prime numbers from 1 to 100.

~~1~~	(2)	(3)	~~4~~	(5)	~~6~~	(7)	~~8~~	~~9~~	~~10~~
11	~~12~~	13	~~14~~	~~15~~	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Prime factorisation

The factors of 24 are: 1, 2, 3, 4, 6, 8 and 12.

Of these factors, 2 and 3 are **prime numbers**. The rest, except 1, are composite numbers.

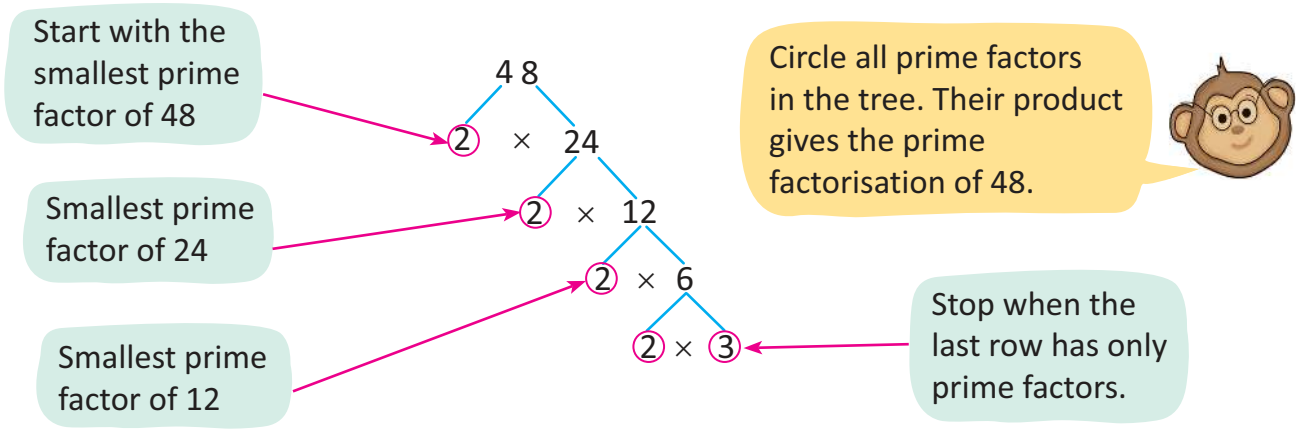
2 and 3 are called the **prime factors** of 24. The number 24 can be written as a product of its prime factors 2 and 3.

$24 = 2 \times 2 \times 2 \times 3$

When a number is expressed as a product of its prime factors, it is called **prime factorisation** of the number. So $24 = 2 \times 2 \times 2 \times 3$ is the prime factorisation of 24.

Finding prime factors of a number

Factor tree method: To find the prime factors of 48, make the factor tree of 48. Continue to factorise until all factors are prime.



Prime factorisation of 48 = 2 × 2 × 2 × 2 × 3

Division method: To find the prime factors of 60:

- Divide by the first prime factor 2. Go on dividing by 2 until the quotient is no longer divisible by 2.
- Then divide by the next prime number, and so on.
- Continue until the quotient is a prime number.

2	60
2	30
3	15
	5

Prime factorisation of 60 = 2 × 2 × 3 × 5

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EXERCISE 3

1. Find the prime factors by constructing factor trees.

a) 32 b) 45 c) 24 d) 63 e) 81

2. Find the prime factors by the division method.

a) 36 b) 75 c) 91 d) 28 e) 88

Highest common factor (HCF)

The factors of 12 are 1, 2, 3, 4, 6 and 12.

The factors of 16 are 1, 2, 4, 8 and 16.

The common factors of 12 and 16 are 1, 2 and 4.

(Refer Maths Lab Activity on page 112.)

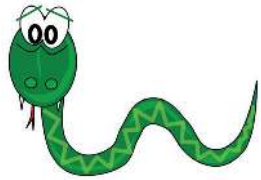
The **highest common factor (HCF)** of 12 and 16 is **4**. It divides both 12 and 16 without leaving a remainder.

The HCF of two numbers is the greatest number that divides the two numbers without leaving a remainder.

You can find the HCF using any of the following methods.

Factor method

Example 1: Find the HCF of 12 and 18.



Factors of 12: 1, 2, 3, 4, 6, 12

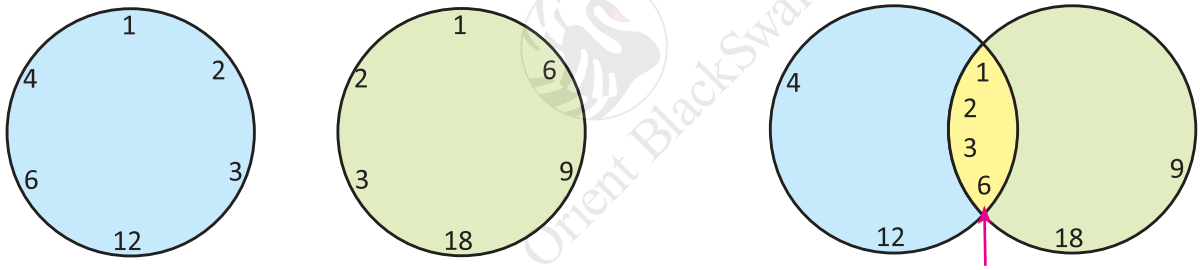
Factors of 18: 1, 2, 3, 6, 9, 18

Common factors = 1, 2, 3, 6

Therefore HCF = 6

Answer: 6

The common factors of 12 and 18 can be shown in a diagram.



Factors of 12 | Factors of 18 | Common factors of 12 and 18

Prime factorisation method

Example 2: Find the HCF of 20 and 30.

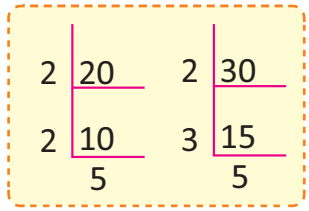
Step 1: Use the division method to find the prime factors.

Step 2: Find the common prime factors.

Prime factorisation of 20 = 2 × 2 × 5

Prime factorisation of 30 = 2 × 3 × 5

Common prime factors = 2, 5



Step 3: Find the product of the common prime factors to get the HCF.

HCF of 20 and 30 = 2 × 5 = 10

Answer: 10

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Example 3: Find the HCF of 12, 18 and 25.

Prime factorisation of 12 = 2 × 2 × 3

Prime factorisation of 18 = 2 × 3 × 3

Prime factorisation of 25 = 5 × 5

There are no common prime factors.

Therefore the HCF = 1

Answer: 1

Whenever there are no common factors between two or more numbers, the HCF is 1.

Two numbers that have only 1 as a common factor are called **coprime numbers**. The HCF of coprime numbers is 1.

Check it!

Question: Find the HCF of 12, 36 and 24.

Check the answers of Tolu, Golu and Mili. Put ✓ for correct and ✗ for wrong. Can you say what the mistakes are?

Tolu	Golu	Mili
12 = 2 × 2 × 3	12 = 2 × 2 × 3	12 = 2 × 2 × 3
36 = 2 × 2 × 3 × 3	36 = 2 × 2 × 3 × 3	36 = 2 × 2 × 3 × 3
24 = 2 × 2 × 2 × 3	24 = 2 × 2 × 2 × 3	24 = 2 × 2 × 2 × 3
HCF = 2 × 3 = 6	HCF = 2 × 2 × 3 = 12	HCF = 2 × 2 × 3 × 3 = 24

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EXERCISE 4

1. Find the common factors and the HCF.

a) 8, 16

Factors of 8: ___, ___, ___, ___

Factors of 16: ___, ___, ___, ___, ___

Common factors: ___, ___, ___, ___

HCF = ____

b) 14, 21

Factors of 14: ___, ___, ___, ___

Factors of 21: ___, ___, ___, ___

Common factors: ___, ___

HCF = _____

2. Fill in the prime factors and find the HCF.

a) 6, 18

6 = 2 × ___

18 = 2 × ___ × ___

HCF =

b) 9, 15

9 = ___ × ___

15 = ___ × ___

HCF =

3. Find the HCF by finding all factors.

a) 20, 30 b) 27, 54 c) 24, 36 d) 25, 40

e) 20, 50, 90 f) 22, 33, 44 g) 14, 21, 28 h) 45, 65, 75

4. Find the HCF by the prime factorisation method.

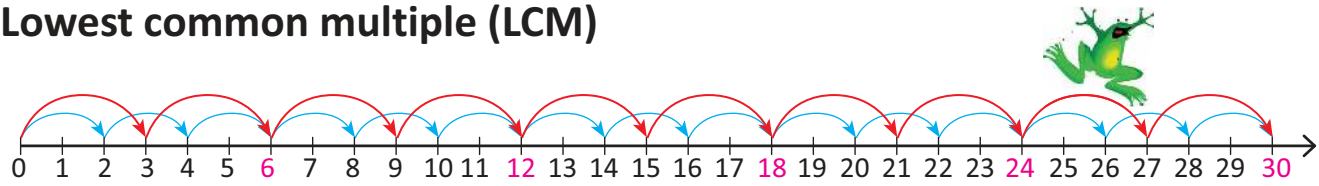
a) 10, 6 b) 15, 25 c) 16, 24 d) 14, 22

e) 49, 77 f) 20, 48 g) 72, 90 h) 40, 56

5. Find the greatest number that divides 56 and 84 exactly.

6. Find the greatest number that divides 28 and 49 without leaving a remainder.

Lowest common multiple (LCM)



The number line shows the multiples of 2 and 3. The numbers where the jumps of 2 and 3 meet are the common multiples of 2 and 3.

The common multiples of 2 and 3 are: 6, 12, 18, 24, 30,...

The **lowest common multiple (LCM)** of 2 and 3 is 6.

The **LCM** of two numbers is the smallest number that can be divided by the two numbers without leaving a remainder.

You can use either of the following two methods for finding LCM.

Prime factorisation method

Example 1: Find the LCM of 12 and 30.

Step 1: Find the prime factorisation of 12.

Step 2: Find the prime factorisation of 30.

Step 3: Multiply the factorisation by considering the common factors 2 and 3 only once.

$12 = 2 \times 2 \times 3$

$30 = 2 \times 3 \times 5$

$LCM = 2 \times 2 \times 3 \times 5 = 60$

$LCM = 60$

Check: $60 \div 12 = 5$; $60 \div 30 = 2$

Answer: LCM of 12 and 30 = 60

Example 2: Find the LCM of 3 and 8.

3 and 8 are coprime numbers.

Prime factorisation of 3 = 3

Prime factorisation of 8 = $2 \times 2 \times 2$

LCM = $3 \times 2 \times 2 \times 2 = 24$ (Note that: $3 \times 8 = 24$)

Answer: LCM of 3 and 8 = 24

Rapid Check

Find the LCM of

i. 15 and 25

ii. 9 and 16

The LCM of coprime numbers is the product of the two numbers.

Division method

Example 1: Find the LCM of 10, 15 and 20.

Step 1: Write the numbers as shown.

Step 2: Divide by a prime number that divides at least two numbers exactly.

Write the quotients below the numbers that are divisible. For numbers that cannot be divided exactly, copy the numbers.

2	10, 15, 20
5	5, 15, 10
	1, 3, 2

LCM = $2 \times 5 \times 3 \times 2 = 60$

Step 3: Keep dividing by prime numbers until the last row has only prime numbers or 1

Step 4: Multiply all divisors, and all prime numbers left in the last row.

LCM = $2 \times 5 \times 3 \times 2 = 60$

Check: $60 \div 10 = 6$; $60 \div 15 = 4$; $60 \div 20 = 3$

Answer: LCM of 10, 15 and 20 = 60

Example 2: Find the LCM of 14, 21 and 42.

2	14, 21, 42
7	7, 21, 21
3	1, 3, 3
	1, 1, 1

LCM = $2 \times 7 \times 3 = 42$

Answer: LCM of 14, 21 and 42 = 42

Rapid Check

Find the LCM of 6, 9 and 12 by the:

- Prime factorisation method
- Division method

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Check it!

Question: Find the LCM of 12 and 38.

Check the answers of Golu and Molu. Put ✓ for correct and ✗ for wrong. Can you say what the mistake is?

Golu

$12 = 2 \times 2 \times 3$

$18 = 2 \times 3 \times 3$

$LCM = 2 \times 2 \times 2 \times 3 \times 3 \times 3 = 216$

Molu

$12 = 2 \times 2 \times 3$

$18 = 2 \times 3 \times 3$

$LCM = 2 \times 2 \times 3 \times 3 = 36$

Relation between HCF and LCM

Complete the table.

Numbers	HCF	LCM	Product of numbers	Product of HCF and LCM
10, 15	5	30	150	150
4, 6	2	12		
6, 9				
9, 15				

What do you notice?

Product of two numbers = Product of their HCF and LCM

EXERCISE 5

1. Find the multiples, common multiples and the LCM.

	Numbers	First 6 Multiples	Common multiples	LCM
a)	3 4	3, 6, 9, 12, 15, 18 4, 8, 12, 16, 20, 24		
b)	6 9			
c)	4 8			
d)	3 6			

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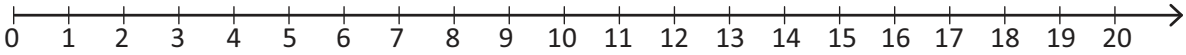
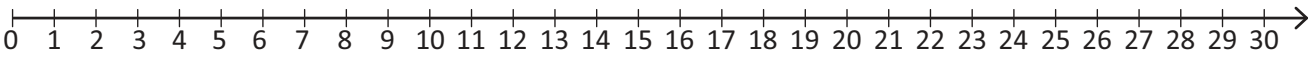
2. Find the LCM on the number line.

a) 2, 5



b) 3, 4



3. Find the LCM by the prime factorisation method.

a) 16, 20 b) 7, 8 c) 3, 7 d) 12, 15

e) 15, 20 f) 21, 28 g) 30, 45 h) 12, 18

4. Find the LCM by the division method.

a) 21, 28 b) 42, 56 c) 24, 36 d) 24, 32

e) 10, 15, 18 f) 12, 15, 40 g) 21, 63, 84 h) 63, 27, 54

5. Fill in the yellow boxes in the table, using the relation between HCF and LCM.

	First number	Second number	HCF of the numbers	LCM of the numbers
a)	12		4	48
b)	15	20	5	
c)		14	7	42
d)	9	15		45

6. Find the smallest number which when divided by 12 and 20 leaves no remainder.

7. Find the smallest number which is exactly divisible by 20, 25 and 30.

Mixed Bag

(Concept, skill, application and thinking based)

1. Choose the correct answer.

a) The prime factorisation of 36 is:

i. $2 \times 3 \times 2 \times 3$ ii. $2 \times 2 \times 3 \times 3$ iii. $3 \times 3 \times 2 \times 2$ iv. All of these

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b) Two numbers 71 and 13 have no common factors. Their LCM is:

i. 71 + 13 ii. 71 × 13 iii. 71 – 13 iv. 71 ÷ 13

c) The product of the HCF and LCM of two numbers is 48. If one of the numbers is 6, what is the other number?

i. 8 ii. 4 iii. 6 iv. 12

d) If a number is divisible by 16, it is also divisible by:

i. 2 ii. 4 iii. 8 iv. All of these

2. Put ✓ if divisible and ✗ if not divisible.

	Divisible by						
Number	2	3	4	5	6	9	10
a) 95							
b) 126							
c) 275							
d) 996							
e) 1350							
f) 6225							
g) 66,660							
h) 54,795							

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3. Find the HCF.

a) 16, 36 b) 12, 32 c) 18, 45 d) 27, 35 e) 27, 36

f) 15, 25, 35 g) 14, 30, 56 h) 20, 30, 40 i) 12, 48, 72 j) 42, 63, 84

4. Find the LCM.

a) 6, 15 b) 12, 16 c) 18, 24 d) 8, 18 e) 21, 25

f) 24, 72, 90 g) 18, 24, 32 h) 14, 28, 30 i) 24, 36, 90 j) 12, 15, 30

5. The product of two numbers is 300. If their LCM is 60, find their HCF.

6. The product of the HCF and LCM of two numbers is 726. If one of the numbers is 22, what is the other number?

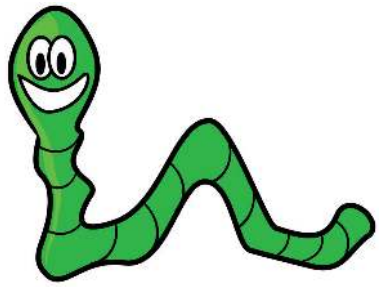
7. Find the largest number that divides 28 and 49 without a remainder.

8. Find the largest number that divides 10 and 14, leaving a remainder of 2.
(*Hint:* If you subtract 2 from both 10 and 14. the numbers you get will be exactly divisible by a few factors. Find their HCF.)

9. Find the smallest number which when divided by 10 and 15, leaves no remainder.

10. Find the smallest number, which when divided by 12 and 15, leaves a remainder of 5.
(*Hint:* The LCM of 12 and 15 will not leave a remainder when divided by 12 and 15. What about LCM + 5?)

Mental Maths



FA

1. What are the first four odd multiples of 5?
2. What are the prime factors of 24?
3. What are the prime factors of 17?
4. What is the smallest number you should add to 98 to make it a multiple of 5?
5. What is the smallest number you should subtract from 286 to make it a multiple of 5?
6. What is the smallest number you should add to 236 to make it divisible by 3?
7. What is the smallest number you should subtract from 456 to make it divisibly by 9?
8. You know that the HCF of coprime numbers is 1. Their LCM = product of the numbers. Find the HCF and LCM of these numbers.
   a) 4, 5 b) 6, 11 c) 5, 9 d) 10, 13
9. The HCF of 4 and 12 is 4. Their LCM is 12.

If one number is a multiple of another, their HCF is the smaller number and the LCM is the bigger number.

Find the HCF and LCM of these numbers.

a) 5, 15 b) 7, 14 c) 4, 16 d) 6, 18

10. The HCF of coprime numbers 5 and 7 is 1. What is the HCF of 50 and 70?

Hots

FA

1. Are all numbers divisible by 10 also divisible by 5? Are all numbers divisible by 5 also divisible by 10?

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2. What is the **lowest** common multiple of 8 and 10? Can you find the **highest** common multiple of 8 and 10? Give reason.
3. How many multiples of 6 are there between 0 and 100? ***(Hint: Divide 100 by 6. What is the quotient?)***

Life Skill

FA

Value-based question

There are 34 children in Simon's class (including him). Simon had a bag with 80 sweets. He gave his classmates 2 sweets each (he also took 2 himself).

Does he now have enough sweets left to divide equally among his 4 best friends in his colony?

Problem Solving

FA

Make your own divisibility rule

Make rules for divisibility by 15 and 18. (Hint: $15 = 5 \times 3$; $18 = 9 \times 2$; there will be two conditions to check in each rule.)

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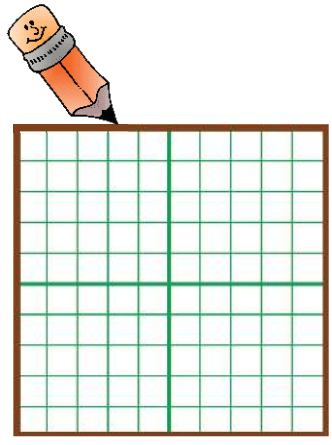
Maths Lab Activity

Activity 1

Objective: To understand the concept of factors

Material required: Centimetre graph papers, pencils

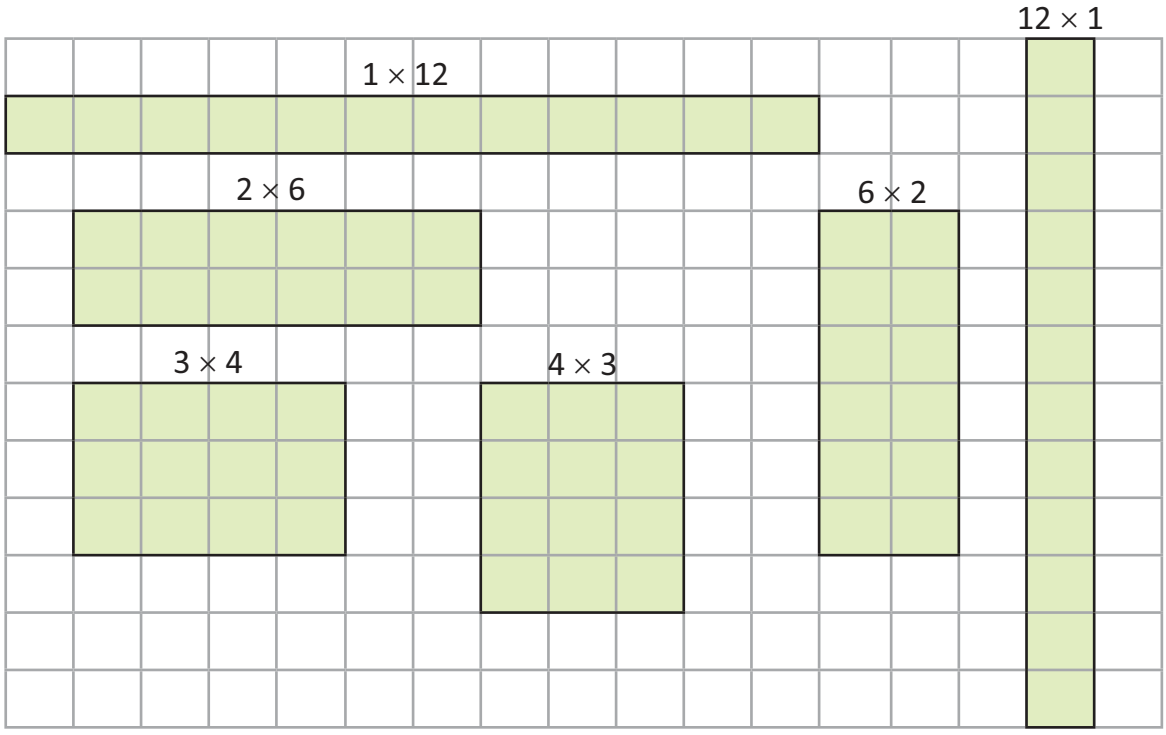


Method: Give pieces of centimetre graph paper to each student. Instruct them to draw all possible rectangles enclosing 12 squares—they can draw squares with dimensions of 1×12, 2×6 and so on.

Let them list the dimensions of the different rectangles drawn. These will give the factors of 12.

Similarly try out with other numbers such as 8, 14, 15 and 20.

Let them try with prime numbers such as 5, 7 and 11. What is their conclusion?



Activity 2

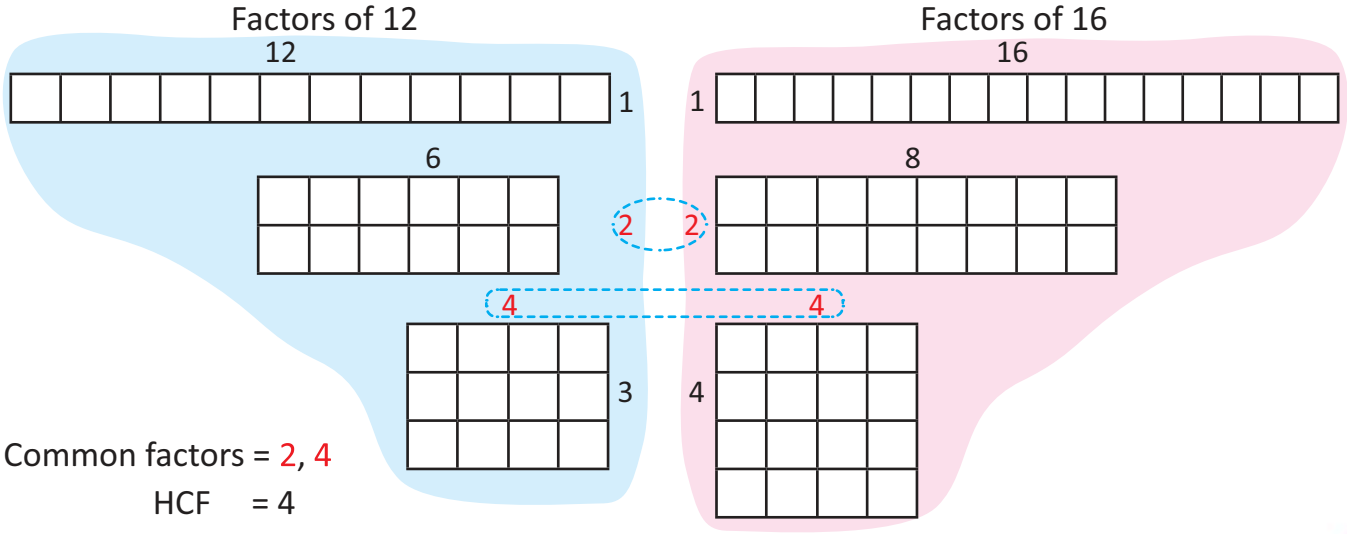
Objective: To find common factors and the HCF using graph paper

Material required: Centimetre graph papers, pencils

Method: Give pieces of centimetre graph paper to each student. Let them work in pairs. To find the common factors and HCF of 12 and 16, one student in a pair draws rectangles to find the factors of 12 (as in Activity 1) and the other student draws rectangles to find the factors of 16.

Then both students compare the sides of the two sets of rectangles to find the common factors. Equal sides show that the factor is common.

They list out all common factors and then find the highest common factor.



Assessment Corner 1 FA

A. Rewrite the numbers by inserting commas according to the Indian and the international system of numeration.

Number	Indian system	International system
64578776		
40987961		
7123442		
1456586		
32544566		

B. Activity

Tick the boxes that are the factors of the given numbers. Then answer the questions given below.

Factors / Numbers	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
11																				
12																				
13																				
14																				
15																				
16																				
17																				
18																				
19																				
20																				

1. Are the factors of a number ‘greater than’, ‘less than’ or ‘equal to’ the number itself?
2. Which number is the factor of all the numbers?
3. How many factors do prime numbers have?
4. Do composite numbers have more factors or fewer factors than prime numbers?

Orient BlackSwan